# Richard W. Hamming

# Learning to Learn

## The Art of Doing Science and Engineering

# Session 11: Coding Theory II

# Coding Theory II Overview

## Two types of Coding

- Source Encoding
- Channel Encoding

## How do we find the best source encoding?

- Huffman encoding

## How do we encode for a noisy channel?

- Error correcting codes

# The Best Source Encoding

We know efficient codes are related to probabilities.

Intuition leads to the solution:

$$p_1 \geq p_2 \geq p_3 \ldots \geq p_n$$
$$l_1 \leq l_2 \leq l_3 \ldots \leq l_n$$

Prove this by exchanging any two lengths and the average code length decreases.

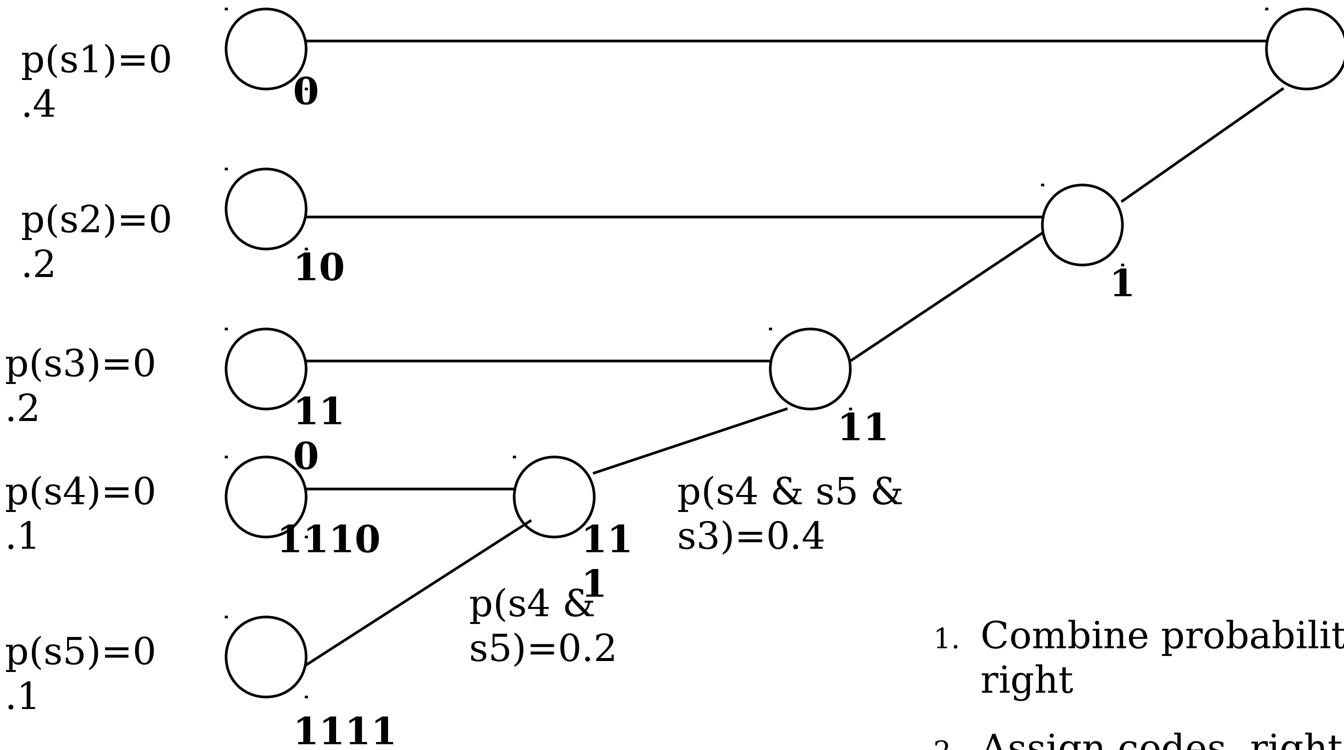
# Deriving Huffman Encoding

**Create the best encoding of symbols from their probabilities.**

**Build encoding tree from bottom up.**

- Merge 2 least frequent nodes, combine their probabilities, repeat.
- Assign 0,1 to final 2 symbols.
- Back down the tree, split symbols, assigning 0 or 1 appended to parent's code.

# Deriving Huffman Encoding Tree (Visual)




1. Combine probabilities, left to right
2. Assign codes, right to left.

# Optimality

**Huffman trees give optimal (most efficient) codes.**

**Do we necessarily want optimal?**

**What characteristics do we want?**

- Efficiency (average code length)?
- Efficiency and low variability?

**Start from an optimum and move one way or the other.**

- Sometimes you can get a large change a desired characteristic but pay little in performance.

# Why Huffman?

**Write a program to do it.**

**Sending Huffman tree and encoded info saves channel or storage capacity.**

**Best possible data compression.**

- If anything can be gained by compression.
- With uniform distribution, thus block code, no compression possible.

# Channel Encoding

## What to do with noise?

- Error detection

## Extra parity bit catches only 1 error.

## Probability of 2 errors in a message of length *n* increases with *n*.

- In a longer message parity bit takes up less channel capacity.
- In a longer message 2nd error more likely.
- Depends on probability *p* of error in a single bit.

# Errors

**Probability of 2,3,4, and higher errors depends mostly on *np* product.**

- *np* near 1 makes errors highly likely.
- Very small *np* makes double, triple errors highly unlikely.

**Tradeoff between error and efficiency.**

**Engineering judgment that requires a concrete case and a real *n* and *p*.**

# Error Handling

## What if an error occurs?

## Repeat the message

- On marginal errors repeat messages will be clear
- On systematic errors you will repeat error until you get a double error.  Not a good strategy.

## Strategy depends on the nature of the errors you expect.

# Detecting Human Error

**Encoding alphanumeric symbols.**

**Humans make different types of errors than machines**

- Changing one digit: 667 to 677
- Exchanging digits: 67 to 76
- Parity can’t cope with the two digit exchange error

# Detecting Human Errors

## Weighted code

- Add 1 parity symbol to a message of length *n-1* so the message checksum below is 0.

$x_k$ is the value for the $k^{th}$ symbol of the message, $0 \le x_k \le 36$

$$\left(\sum_{k=1}^{n} k \cdot x_k\right) \bmod 37$$

# Different Types of Codes

- ISBN is a weighted code with modulo 11.
- Even/odd parity bit is for white noise
- Weighted code is for human noise.
  - But machine can easily check code for mistake
- Many types of codes not talked about.
- Know what the noise is to determine type of code.
- Design a code to meet the cope with your particular type of noise problem.

# Creativity in Deriving Codes

- Learn principles, then solve your problem.
- Next lecture will tell how Hamming derived his codes.
- If you were in the same situation Huffman or Hamming were in, could you come up with the codes if you had to?
- Creativity is simple if you are prepared.

# Parting Thoughts

**Many scientists wasted the second half of their lives elaborating a famous theory.**

- Leave elaborations to others.
- Follows from Hamming's earlier statement, that those who come up with great ideas often don't understand their own ideas as well as do those who follow.

**Set great work aside, try to do something else.**